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Properties of Alfven Waves with Transverse Scale on the Order of Skin-Depth

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OVERVIEW

It is widely perceived that shear Alfven waves propagate electromagnetic disturbances strictly along the ambient magnetic field. This feature is a consequence of the neglect of the parallel electric field, an approximation appropriate for large perpendicular wavelengths ($k_\perp \to 0$). However, for small transverse scales, the parallel electron currents cannot be neglected.

This analytical study examines shear Alfven waves whose scale across the magnetic field is on the order of the electron skin-depth $c/\omega_{pe} \equiv k_s^{-1}$. This physical situation is motivated by the interest in spontaneous filamentation in magnetized plasmas, studies of axial relaxation of transport currents, and models of electron acceleration in space plasmas. At the basic level, this study addresses how a microscopic current channel spreads.

It is found that Alfven wave patterns exhibit a collisionless divergence determined by propagation cones that emanate from the edges of the exciting structures. The spreading cones follow an angle θ relative to the magnetic field given by $\tan\theta = (\omega/\Omega_i)\,(m/M)^{1/2}\,[1-(\omega/\Omega_i)^2]^{-1/2}$, where ω is the wave frequency, Ω_i is the ion gyrofrequency, m and M are the electron and ion mass. Within a few wavelengths of the exciter, a radial diffraction pattern develops due to coaxial currents induced by the skin effect. The transition from collisionless divergence along cone trajectories to radial diffusion occurs for values of ν_e/ω slightly larger than unity, where ν_e is the electron collision frequency.

Recent experiments [1] in the LAPD facility at UCLA have verified several of the theoretical predictions of this study.

FORMULATION

Using the simplest model of excitation consisting of a disk of radius a, with the ambient magnetic field perpendicular to the disk plane and pointing along the z direction, results in a wave magnetic field

$$\tilde{B}_\theta\,(r,z,\omega) = \frac{2I_o}{ca}\int_0^\infty dk\,\frac{\sin ka}{k}\,J_1(kr)\,\exp\,[ik_{\parallel}(k)z]\,, \tag{1}$$

where $k_{\parallel}(k) = k_A\,[1 + (k/k_s)^2]^{1/2}$ with k_A the Alfven wavenumber at frequency ω, and I_o is the AC current to the disk. The corresponding scaled pattern is shown in Figs. 1 and 2. Collisions are incorporated by including the complex components of the dielectric tensor in k_A. An example of the effects produced is shown in Fig. 3. The diffusive pattern ($\Gamma = 5$) is analogous to the cases investigated by Borg, et al [2].

The effects caused by bulk plasma flow at speed v_D on a shear Alfven wave excited by a strip antenna, infinitely long in the y direction and of width a along x, are described by

$$\tilde{B}_y\,(x,z,\omega) = \frac{2I_o'}{ica}\int_{-\infty}^{\infty} dk\,\frac{\sin\left(\frac{ka}{2}\right)}{k^2}\exp\,\{i\,[kx + k_{\parallel}(k)z]\}\,, \tag{2}$$

in which $k_{\parallel}(k) = k_A\,[1 + (k/k_s)^2]^{1/2}\,(1 - kv_D/\omega)$, and I_o' is the current per unit length delivered to the strip. As seen in Fig. 4 the plasma flow causes a convection of the center of the current channel and induces oscillations that extend beyond the cone trajectories. These features may be of relevance to magnetic turbulence at the edge of magnetized plasmas.

As has been recently emphasized by P.M. Bellan of Caltech, shear Alfven waves have a cut-off at the points $\omega = k_{\|}v_A$, and not a resonance. Consequently, it is possible to encounter various interesting f[illegible] determined by their reflection properties, which are summarized in Fig. 5. A jump in [illegible] in the x direction acts as a high-pass filter for small density increases. However, f[illegible] n > 6, an effective ducting of shear modes occurs. The axial density jump acts as a partial reflector.

The axial electric field of a shear mode propagating in the x - z plane in a plasma whose density varies along the x direction is given by $\tilde{E}_z(x)\exp(ik_{\|}z)$ with $\tilde{E}_z$ determined by

$$\frac{d}{dx}\left\{\frac{\varepsilon_\perp}{\varepsilon_\perp - n_z^2}\frac{d}{dx}\tilde{E}_z\right\} + k_o^2\,\varepsilon_{\|}\tilde{E}_z = 0\,, \tag{3}$$

where $\varepsilon_\perp$, $\varepsilon_{\|}$ are the perpendicular and parallel components of the cold plasma dielectric tensor, $k_o = \omega/c$, $n_z = k_{\|}/k_o$. Near the reflection layer at $\varepsilon_\perp = n_z^2$ the waveform obtained from Eq. (3) is the derivative of the Airy function, i.e., $\tilde{E}_z \sim Ai'(\xi)$ with $\xi = (M/m)^{1/3}(\Omega_i L/c)^{2/3}(x/L)$, and L the scale length. Thus, yielding [illegible] field structure whose width is independent of density and frequency.

SUMMARY

The propagation of small scale shear waves exhibits new features, amenable to experimental study, which may have important consequences for energy transport and particle acceleration.

This work is sponsored by ONR.

References

1. W. Gekelman, et al., UCLA-PPG 1514, May 1994.
2. G.G. Borg, et al., Plasma Phys. and Cont. Fusion 27, 1125 (1985).

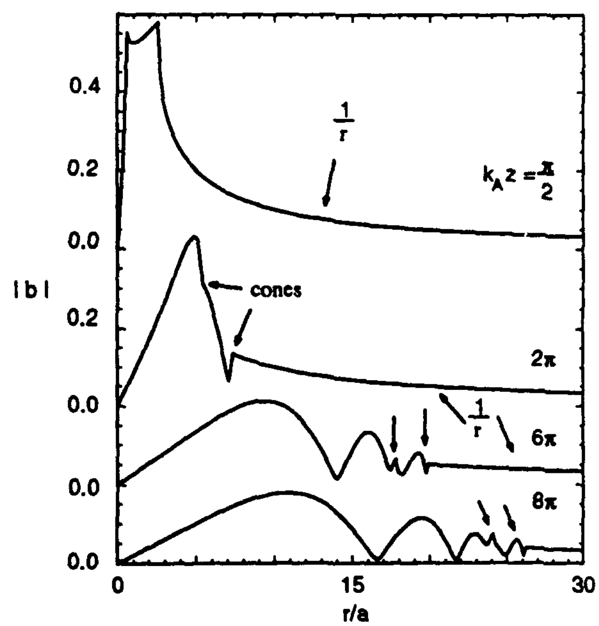

Fig. 1. Radial dependence of scaled magnetic field for different axial positions z. Radius of disk is c/ω_{pe}. Axial current channel spreads up to the cone trajectories. Resistive diffusion allows expansion beyond cones, as shown in Fig. 3.

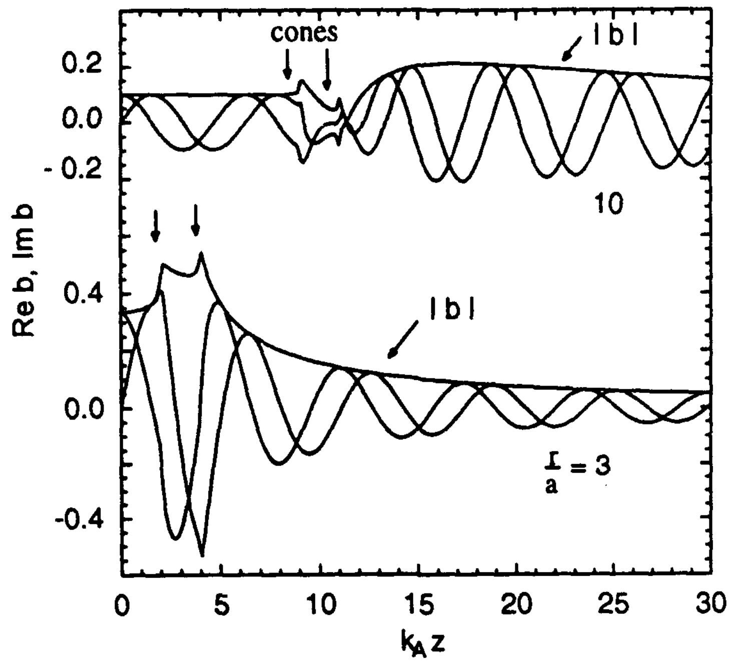

Fig. 2. Axial dependence of the real and imaginary parts of the scaled magnetic field for two radial positions outside the disk shadow. Axial decay at r/a = 3 corresponds to radial spreading seen in Fig. 1.

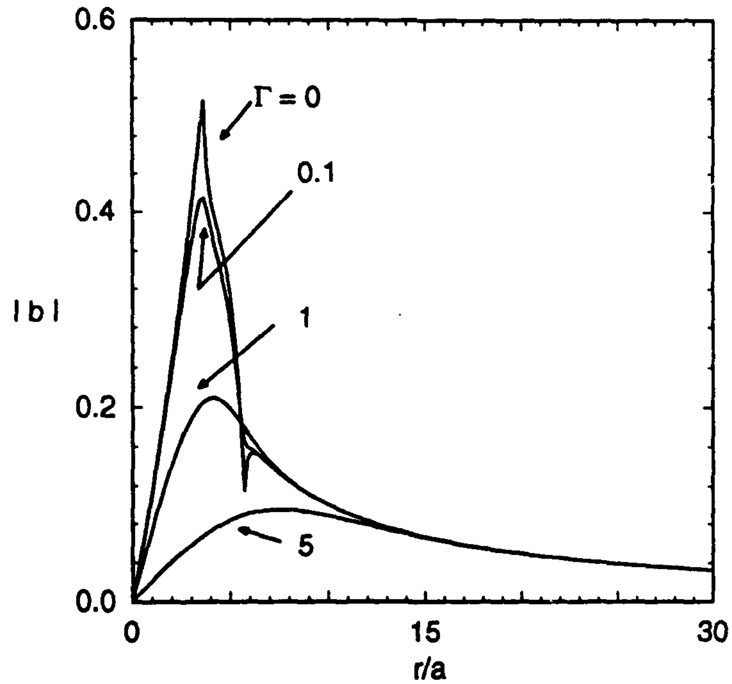

Fig. 3. Transition from collisionless divergence set by cone trajectory to diffusive spread as collision frequency increases. $\Gamma = \nu_e/\omega$, $k_A z = 3\pi/2$.

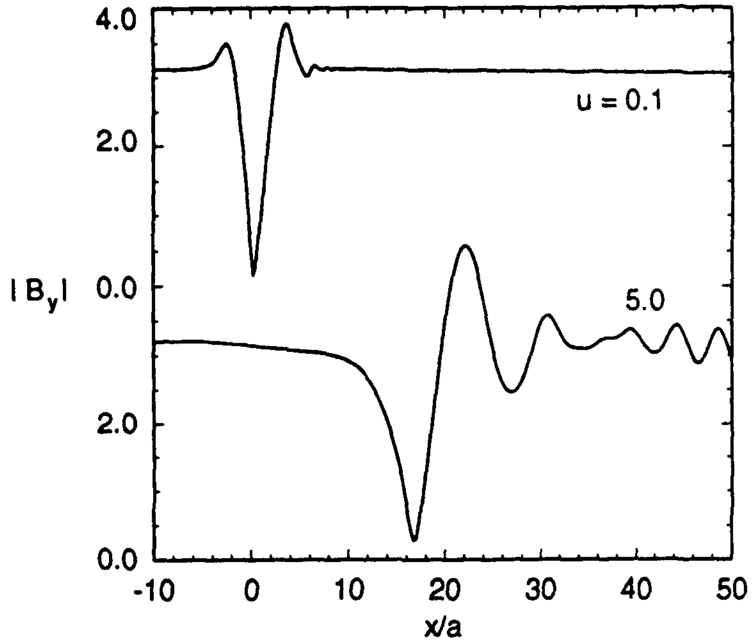

Fig. 4. Transverse dependence of magnitude of scaled magnetic field at axial position $k_A z = \pi$ for two values of scaled drift $u = 2v_D/a\omega$ along x. B_y is constant outside axial current channel for this geometry. Antenna width is $a = c/\omega_{pe}$.

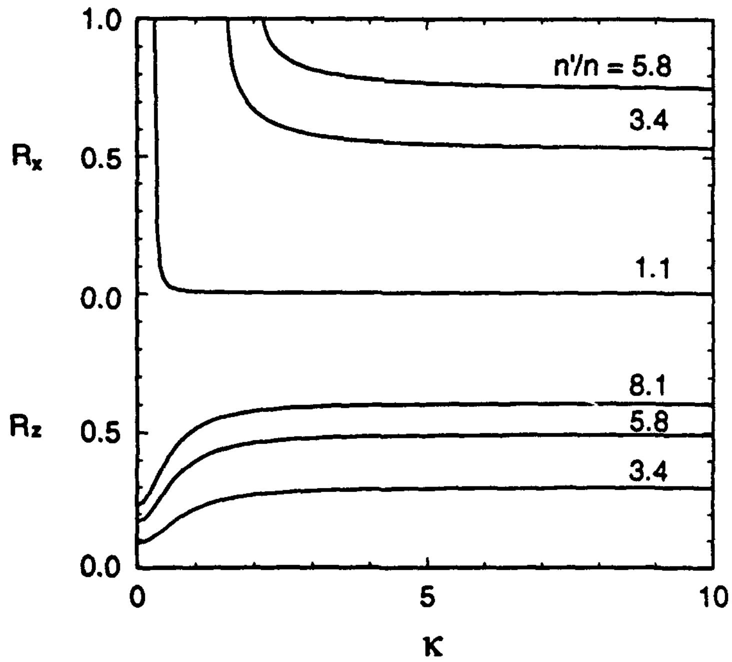

Fig. 5. Dependence of reflection coefficient on scaled transverse wave number $\kappa = k_\perp/k_s$, for a density jump n'/n transverse to the ambient field, R_x, and along the field, R_z.